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WEAK SHOCK WAVES IN THE IONOSPHERE

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Abstract

It is shown that in the D-region of the ionosphere, the shock formed by the nose of a sounding rocket introduces serious errors in measurement of the ionic composition of the region. A simple model of the chemistry of the ionosphere is developed, which correctly predicts the equilibrium composition of the D-region as estimated by Nicolet and Aikin[1]. Using this model, we made a calculation of the chemical structure of a weak shock in the ionosphere. The result is compared with an equilibrium calculation. The effects of impurities are discussed. Author

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Weak Shock Waves in the Ionosphere

I. Introduction

The D-region of the ionosphere lies between about 50 and 90 km. above the earth's surface. The principal constituents of the atmosphere in this region are N_2 and 0_2. The ionic constituents of this region, which are formed by photoionization, have been estimated by Nicolet and Aikin(1), Nicolet and Swider(2) and others (see ref. 2). Nicolet and Aikin(1) find the dominant positive ions to be A_2^+. (A stands for either N or 0, so that A_2^+ signifies the chemically similar ions $N0^+$, N_2^+, and 0_2^+). As the altitude decreases from 70 km. the ratio of negative ions to electrons increases reaching 10 at 50 km.

The number of neutrals varies roughly exponentially from $10^{16}/cm^3$ at 50 km. to $10^{13}/cm^3$ at 90 km(3). The positive ion concentration is about $10^3/cm^3$ and is roughly constant up to 85 km. for a quiet sun.(1) The number of dissociated atoms (A) is about $10^{11}/cm$ and is roughly constant with altitude. The temperature varies from 275^oK at 50 km. to 190^oK at 90 km.

The small concentration of charged particles is important for two reasons. First, the concentration depends rather directly on the spectrum and intensity of the incident radiation. Second, the presence of electrons gives the gas a finite electrical conductivity, , which damps radio waves passing through it. One standard technique to measure the charged particle concentration in the D-region is to mount appropriate instruments on a sounding rocket.

A typical sounding rocket traverses the lower portion of this region at about Mach two. The diameter of a typical sounding rocket might be

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1. Nicolet and Aikin, J. Geophys. Research, 65, 1469 (1960). See also Webber, J. Geophys. Research, 67, 5091 (1962).

2. Nicolet and Swider, Planet. Space Sci., 11, 1459 (1963).

30 cm. As the mean free path of neutrals is 10^{-2}cm. at 50 km., and 10 cm. at 90 km., a weak oblique shock will be formed by the nose of the rocket, up to about 80 km.

We show that such a weak shock can cause a significant percentage change in the ionic composition of the gas behind the shock, with corresponding errors in the data of rocket borne experiments in the D-region. A comparison between equilibrium estimates of this error and the results of a detailed calculation is presented. By studying the chemical structure behind the shock, we show, with certain restrictions, that equilibrium calculations are far too pessimistic.

II. An Equilibrium Calculation

An appropriate reaction to produce ions in the 50-70 km. range is

$$E + 2A_2 \rightleftarrows A_2^+ + A_2^- . \tag{1}$$

E is about 12 ev. for this reaction. From the law of mass action, the mass fraction of positive ions, α, is related to the temperature of the mixture by (K_o is a constant of order one, and k is Boltzman's constant)

$$\frac{\alpha^2}{(1 - 2\alpha)^2} = K_o \exp\left(-\frac{E}{kT}\right). \tag{2}$$

For very low concentrations of ions, it is easy to show that the characteristics of the weak shock are unaffected by the presence of the ions. Thus the shock is an ordinary gas dynamic shock which produces a temperature jump ΔT, with corresponding pressure and density jumps. The percentage change in α is

$$\frac{\Delta\alpha}{\alpha} = \frac{E}{2kT} \quad \frac{\Delta T}{T} \tag{3}$$

$\frac{E}{kT}$ is a large number, about 700 for T = 200°K. For a two dimensional wedge of 5° half angle at Mach two, $\frac{\Delta T}{T}$ = 0.20.[4].

Hence

$$\frac{\Delta\alpha}{\alpha} = 70 . \tag{4}$$

III. A more Detailed Calculation With Finite Reaction Rates

The equilibrium calculation assumes that the reaction (1) is infinitely fast in both directions. However, for nitrogen-oxygen reactions at ionospheric temperatures, endothermic reactions proceed at a negligible rate[2]. This is confirmed using the reaction rates quoted by Lin and Teare[5] and Teare[6], who studied strong shocks in reacting oxygen-nitrogen mixtures.

The ions are actually produced by photoionization from cosmic rays, x-rays and Lyman α[2]. The reaction is

$$A_2 + h\nu \rightarrow A_2^+ + e \tag{5}$$

and the rate for quiet solar conditions is $q_5 = 10^{-18}$/sec at 50 km. to 5×10^{-16}/sec at 90 km. The variation is due to the ionization of nitric oxide by Lyman α which occurs only at high altitudes[2]. These rates may be raised by x-rays by a factor of 10^5 when there are strong solar flares.

The electrons and ions produced by photoionization of atoms are negligible compared to those produced by (5) as the rates are about the same, and the number of atoms is much smaller than the number of molecules.

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4. Liepmann and Roshko, Elements of Gas dynamics, New York, John Wiley, (1957).

5. Lin and Teare, Phys. of Fluids, 6, 355, (1963).

6. Teare, Ionization in High Temperature Gases, Vol. 12 of Progress in Astronautics and Aeronautics, p. 217, Academic Press, (1963).

The photodissociation of molecules is important, however. The process is

$$A_2 + h\nu \rightarrow 2A \qquad (6)$$

Nicolet finds a rate of $q_6 = 10^{-9}/\text{sec}$ for dissociation of oxygen at 75 km[7]

The appropriate recombination process are[2], [5]

$$A_2^+ + e \xrightarrow{k_7} 2A \qquad (7)$$

$$A_2 + 2A \xrightarrow{k_8} A_2 + A_2 \qquad (8)$$

These reactions are faster than radiative recombination. Typical values for the rate coefficients are[5] $k_7 = 10^{-6}\text{cm}^3/\text{sec}$ and $k_8 = 10^{-31}\text{cm}^6/\text{sec}$ at 200°K.

Negative ions are produced by a three body process[2] given by

$$A_2 + A_2 + e \xrightarrow{k_9} A_2^- + A_2 \qquad (9)$$

The rate coefficient is $k_9 = 2.3 \times 10^{-30}\text{cm}^6/\text{sec}$[8]. The photodetachment process for negative ions is faster than recombination.

$$A_2^- \xrightarrow{k_{10}} A_2 + e \qquad (10)$$

Here $k_{10} = 0.44/\text{sec}$[9]

The equations (5) through (10) give correct orders of magnitudes for the chemical structure of the D-region as given in reference 1. Distinguishing between nitrogen and oxygen adds some fifteen or twenty equations to the present six[2], [5]. Here such detail seems unwarranted due to the more important uncertainties in reaction rates and in effects of impurities.

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7. Nicolet, Ionospheric Research Rept. 102, (1958), Penn State U.
8. Chanin, Phelps and Biondi, Phys. Rev., 128, 219 (1962).
9. Burch, Smith and Branscomb, Phys. Rev., 112, 171 (1958).

Examination of the orders of magnitude of the reaction rates times the appropriate number densities yields the following result. Only one of the six reactions is fast enough to cause an appreciable percentage change in composition over a length of the sounding rocket. It is reaction (9). Thus the flow is nearly frozen. The percentage changes in $n(A_2^+)$ and $n(A)$ are caused by the density jump across the shock. For example,

$$\frac{\Delta n(A_2^+)}{n_\infty(A_2^+)} = \frac{\Delta\rho}{\rho} \tag{11}$$

The percentage change in $n(A_2)$ due to negative ion production (reaction 9) can be gotten by writing down the equation for conservation of atoms, but the effect is negligible since $\dfrac{n_\infty(A_2^-)}{n_\infty(A_2)} \ll 1$. Hence the percentage change in $n(A_2)$ is also caused by the density jump across the shock. By conservation of charge the electron concentration is equal to the difference between the positive and negative ion concentrations. The percentage change in electron concentration due to negative ion production cannot be ignored. The rate equation for the number density of A_2^- is[10]

$$\frac{d}{dt}\, n(A_2^-) = n(A_2^-)\,\frac{d}{dt}\,\ln\,\rho + k_9\, n^2(A_2)\, n(e) - k_{10}\, n(A_2^-) \tag{12}$$

where d/dt is the substantial derivative.

We assume that the rate coefficient k_9 varies with temperature as T^{+1} (estimated from fig. 10 of reference (8)). Upstream of the shock, there is an equilibrium described by setting the derivatives equal to zero. Upstream conditions are denoted $(\ )_\infty$. Ignoring the slow reaction (10), and setting

$$\sigma = \frac{\Delta n(A_2^-)}{n_\infty(A_2^-)} - \frac{\Delta\rho}{\rho}$$

10. See ref. 5, for example

the rate equation behind the shock becomes

$$u \frac{d\sigma}{dx} = - k_{9\infty} \; n_\infty^2 (A_2^+) \sigma + k_{9\infty} n_\infty^2 (A_2) \left(\frac{n_\infty (A_2^+)}{n_\infty (A_2^-)} - 1 \right) \left(+ \frac{\Delta T}{T} + \frac{3\Delta \rho}{\rho} \right),$$

which has a solution

$$\frac{\Delta n (A_2^-)}{n_\infty (A_2^-)} = \frac{\Delta\rho}{\rho} + k_{9\infty} \; n_\infty^2 (A_2) \left(\frac{n_\infty (A_2^+)}{n_\infty (A_2^-)} - 1 \right) \left(+ \frac{\Delta T}{T} + \frac{3\Delta\rho}{\rho} \right)$$

$$\left(1 - \exp - \frac{k_{9\infty} \; n_\infty^2 (A_2^+) x}{u} \right) \qquad (13)$$

The boundary condition used to get equation 13

$$\left(\frac{\Delta n (A_2^-)}{n_\infty (A_2^-)} = \frac{\Delta\rho}{\rho} \quad \text{at } x = 0 \right)$$

assumes frozen flow through the shock. This is admissible since at 50 km. the reaction takes about 10^4 mean free paths to be completed.[11] Here x is the distance downstream from the shock, and u the velocity behind the shock. For a wedge of 5° half angle at Mach two

$$\frac{\Delta\rho}{\rho} = .50 \qquad (14)$$

Hence ignoring the shock gives a 50% error in percentage composition just behind the shock. At 50 km., 10 centimeters behind the shock, the second term in equation (13) adds 12%. Due to the exponential decrease in the number of molecules with increasing altitude, this term is negligible above 70 km.

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11. Weak shocks are one mean free path thick (See ref. 4).

IV. Conclusions

Comparing the results of section II and III, we may conclude with one reservation, that the flow behind a shock formed by a sounding rocket in the D-region is nearly frozen. There can be sizeable errors in percentage composition introduced by ignoring the density of jump across the shock.

The reservation concerns the role of impurities. For example, sodium is easily ionized, with a photoionization rate of 10^{-7} sec[1]. In the unlikely event that a reaction of the type

$$N_a^{+} + A_2 + \text{about 1 ev.} \xrightarrow{k} N_a + A_2^{+} \tag{15}$$

has an appreciable rate under ionospheric conditions, the change in rate constant going through the shock would be large, due to the exponential dependence of k on temperature. The percentage change in k would be

$$\frac{\Delta k}{k} = \frac{E}{kT} \frac{\Delta T}{T} \tag{16}$$

For a wedge of 5^{o} half angle at Mach two, and E = 1 ev. at 200^{o}K,

$$\frac{\Delta k}{k} \doteq 11 \tag{17}$$

Hence the effects of impurities could raise the error estimated in section III by an order of magnitude.

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